10 Division of Polynomials

Let's recall.

Last year we have studied how to perform addition, subtraction and multiplication on algebraic expressions.

Fill in the blanks in the following examples.

(1) $2a + 3a = \square$ (2) $7b - 4b = \square$

(3) $3p \times p^2 = \square$ (4) $5m^2 \times 3m^2 = \square$

(5) $(2x + 5y) \times \frac{3}{x} = \square$ (6) $(3x^2 + 4y) \times (2x + 3y) = \square$

Let's learn.

Introduction to polynomial

If index of each term of an algebraic expression in one variable is a whole number, then the expression is called a polynamial.

For example, $x^2 + 2x + 3$; $3y^3 + 2y^2 + y + 5$ are polynamials in one variable.

Polynomials are specific algebraic expressions. Hence the operations of addition, subtraction and multiplication on polynomials are similar to those performed on algebraic expressions.

Ex. (1) $(3x^2 - 2x) \times (4x^3 - 3x^2)$

$= 3x^2(4x^3 - 3x^2) - 2x(4x^3 - 3x^2)$

$= 12x^5 - 9x^4 - 8x^4 + 6x^3$

$= 12x^5 - 17x^4 + 6x^3$

(2) $(4x - 5) - (3x^2 - 7x + 8)$

$= 4x - 5 - 3x^2 + 7x - 8$

$= -3x^2 + 4x + 7x - 8$

$= -3x^2 + 11x - 13$

Degree of a polynomial

Write the greatest index of the variable in the following polynomials.

Ex. (1) The greatest index of variable in the polynomial $3x^2 + 4x$ is $\boxed{2}$

Ex. (2) The greatest index of a variable in the polynomial

$7x^3 + 5x + 4x^5 + 2x^2$ is $\square$

The greatest index of the variable in the given polynomial is called the degree of the polynomial.

Now I know.

- The algebraic expression in one variable is a polynomial if the index of variable of each term is a whole number.
- In a polynomial, the greatest index of the variable is the degree of the polynomial.

Let's learn.

(I) To divide a monomial by a monomial

Ex. (1) Divide : $15p^3 \div 3p$

Solution: Division is the opposite operation of multiplication.

For division $15p^3 \div 3p$, we find the multiplier of $3p$ which will give product $15p^3$.

$3p \times 5p^2 = 15p^3 \quad \therefore \quad 15p^3 \div 3p = 5p^2$

$$\begin{array}{r|l} & 5p^2 \\ 3p & 15p^3 \\ & \underline{-15p^3} \\ & 0 \end{array}$$

Ex. (2) Divide and write the correct terms in the boxes.

(i) $(-36x^4) \div (-9x)$ (ii) $(5m^2) \div (-m)$ (iii) $(-20y^5) \div (2y^3)$

$$\begin{array}{r|l} & \boxed{} \\ -9x & -36x^4 \\ & \underline{\boxed{}} \\ & \boxed{} \end{array} \qquad \begin{array}{r|l} & \boxed{} \\ -m & 5m^2 \\ & \underline{\boxed{}} \\ & \boxed{} \end{array} \qquad \begin{array}{r|l} & \boxed{} \\ 2y^3 & -20y^5 \\ & \underline{\boxed{}} \\ & \boxed{} \end{array}$$

To divide a polynomial by a monomial

Study the following examples and learn the method of division of polynomial by a monomial.

Ex. (1) $(6x^3 + 8x^2) \div 2x$

Solution :

$$\begin{array}{r|l} & 3x^2 + 4x \\ 2x & 6x^3 + 8x^2 \\ & \underline{-6x^3} \\ & 0 + 8x^2 \\ & \underline{-8x^2} \\ & 0 \end{array}$$

Explanation –

(i) $2x \times \boxed{3x^2} = 6x^3$

(ii) $2x \times \boxed{4x} = 8x^2$

$\therefore$ Quotient $= 3x^2 + 4x$

Remainder $= 0$

Ex. (2) $(15y^4 + 10y^3 - 3y^2) \div 5y^2$

Solution :

$$\begin{array}{r|l} & 3y^2 + 2y - \frac{3}{5} \\ \hline 5y^2 & 15y^4 + 10y^3 - 3y^2 \\ & \underline{-15y^4} \\ & 0 + 10y^3 - 3y^2 \\ & \underline{-10y^3} \\ & 0 - 3y^2 \\ & \underline{\mp 3y^2} \\ & 0 \end{array}$$

Explanation –

(i) $5y^2 \times \boxed{3y^2} = 15y^4$

(ii) $5y^2 \times \boxed{2y} = 10y^3$

(iii) $5y^2 \times \boxed{\frac{-3}{5}} = -3y^2$

$\therefore$ Quotient $= 3y^2 + 2y - \frac{3}{5}$ Remainder $= 0$

Ex. (3) $(12p^3 - 6p^2 + 4p) \div 3p^2$

Solution :

$$\begin{array}{r|l} & 4p - 2 \\ \hline 3p^2 & 12p^3 - 6p^2 + 4p \\ & \underline{-12p^3} \\ & 0 - 6p^2 + 4p \\ & \underline{\mp 6p^2} \\ & 0 + 4p \end{array}$$

Explanation –

(i) $3p^2 \times \boxed{4p} = 12p^3$

(ii) $3p^2 \times \boxed{-2} = -6p^2$

$\therefore$ Quotient $= 4p - 2$

Remainder $= 4p$

Ex. (4) $(5x^4 - 3x^3 + 4x^2 + 2x - 6) \div x^2$

Solution :

$$\begin{array}{r|l} & 5x^2 - 3x + 4 \\ \hline x^2 & 5x^4 - 3x^3 + 4x^2 + 2x - 6 \\ & \underline{-5x^4} \\ & 0 - 3x^3 + 4x^2 + 2x - 6 \\ & \underline{\mp 3x^3} \\ & 0 + 4x^2 + 2x - 6 \\ & \underline{-4x^2} \\ & 0 + 2x - 6 \end{array}$$

Explanation –

(i) $x^2 \times \boxed{5x^2} = 5x^4$

(ii) $x^2 \times \boxed{-3x} = -3x^3$

(iii) $x^2 \times \boxed{4} = 4x^2$

$\therefore$ Quotient $= 5x^2 - 3x + 4$ Remainder $= 2x - 6$

While dividing a polynomial, the operation of division ends when either the remainder is zero or the degree of the remainder is less than the degree of the division polynomial.

Note that, in the above example (3), the degree of remainder $4p$ is less than the degree of the division $3p^2$. Similarly in the example (4), the degree of remainder $(2x - 6)$ is less than the degree of the division polynomial x^2.

Practice Set 10.1

1. Divide. Write the quotient and the remainder.

(1) $21m^2 \div 7m$ (2) $40a^3 \div (-10a)$

(3) $(-48p^4) \div (-9p^2)$ (4) $40m^5 \div 30m^3$

(5) $(5x^3 - 3x^2) \div x^2$ (6) $(8p^3 - 4p^2) \div 2p^2$

(7) $(2y^3 + 4y^2 + 3) \div 2y^2$ (8) $(21x^4 - 14x^2 + 7x) \div 7x^3$

(9) $(6x^5 - 4x^4 + 8x^3 + 2x^2) \div 2x^2$ (10) $(25m^4 - 15m^3 + 10m + 8) \div 5m^3$

Let's learn.

To divide a polynomial by a binomial

The method of division of a polynomial by a binomial is the same as the division of a polynomial by a monomial.

Ex. (1) $(x^2 + 4x + 4) \div (x + 2)$

$$\begin{array}{r|l} & x + 2 \\ \hline x + 2 & x^2 + 4x + 4 \\ & \underline{\overset{-}{}x^2 \underset{-}{+} 2x} \\ & 0 + 2x + 4 \\ & \underline{\underset{-}{+} 2x \underset{-}{+} 4} \\ & 0 \end{array}$$

Solution :

Explanation

(i) First, write the dividend and division in the descending order of their indices. Multiplying the first term of division by x, we get first term of the dividend.

$\therefore$ multiply the division by x

(ii) $(x + 2) \times \boxed{2} = 2x + 4$

$\therefore$ Quotient $= x + 2$; Remainder $= 0$

Ex. (2) $(y^4 + 24y - 10y^2) \div (y + 4)$

Solution: In this example, degree of the dividend polynomial is 4. The indices of variable in it are not in descending order. The term with index 3 is missing. Assume it as $0y^3$. Write the dividend in the descending order of indices and then divide.

$$
\begin{array}{r|l}
 & y^3 - 4y^2 + 6y \\ \hline
y + 4 & y^4 + 0y^3 - 10y^2 + 24y \\
 & \underset{-}{}y^4 \underset{-}{+} 4y^3 \\ \hline
 & 0 - 4y^3 - 10y^2 + 24y \\
 & \underset{+}{-} 4y^3 \underset{+}{-} 16y^2 \\ \hline
 & 0 + 6y^2 + 24y \\
 & \underset{-}{}6y^2 \underset{-}{+} 24y \\ \hline
 & 0
\end{array}
$$

Explanation –

(i) $(y + 4) \times y^3 = y^4 + 4y^3$

(ii) $(y + 4) \times -4y^2 = -4y^3 - 16y^2$

(iii) $(y + 4) \times 6y = 6y^2 + 24y$

$\therefore$ Quotient $= y^3 - 4y^2 + 6y$; Remainder $= 0$

Ex. (3) $(6x^4 + 3x^2 - 9 + 5x + 5x^3) \div (x^2 - 1)$

Solution :

$$
\begin{array}{r|l}
 & 6x^2 + 5x + 9 \\ \hline
x^2 - 1 & 6x^4 + 5x^3 + 3x^2 + 5x - 9 \\
 & \underset{-}{}6x^4 \qquad \underset{+}{-} 6x^2 \\ \hline
 & 0 + 5x^3 + 9x^2 + 5x - 9 \\
 & \underset{-}{+} 5x^3 \qquad \underset{+}{-} 5x \\ \hline
 & 0 + 9x^2 + 10x - 9 \\
 & \underset{-}{}9x^2 \qquad \underset{+}{-} 9 \\ \hline
 & 0 + 10x + 0
\end{array}
$$

Explanation –

(i) $(x^2 - 1) \times 6x^2 = 6x^4 - 6x^2$

(ii) $(x^2 - 1) \times 5x = 5x^3 - 5x$

(iii) $(x^2 - 1) \times 9 = 9x^2 - 9$

$\therefore$ Quotient $= 6x^2 + 5x + 9$; Remainder $= 10x$

Now I know.

- While dividing a polynomial, the operation of division ends if the remainder is zero or the degree of the remainder is less than the degree of the divisior.
- If terms in the dividend are not in descending order, write them in descending order of indices. If any index term is missing, assume the cofficient of that term to be 0 and then complete the descending order.

Practice Set 10.2

1. Divide and write the quotient and the remainder.

(1) $(y^2 + 10y + 24) \div (y + 4)$

(2) $(p^2 + 7p - 5) \div (p + 3)$

(3) $(3x + 2x^2 + 4x^3) \div (x - 4)$

(4) $(2m^3 + m^2 + m + 9) \div (2m - 1)$

(5) $(3x - 3x^2 - 12 + x^4 + x^3) \div (2 + x^2)$

(6★) $(a^4 - a^3 + a^2 - a + 1) \div (a^3 - 2)$

(7★) $(4x^4 - 5x^3 - 7x + 1) \div (4x - 1)$

Answers

Practice Set 10.1

1. $3m$, 0 **2.** $-4a^2$, 0 **3.** $\frac{16}{3}p^2$, 0 **4.** $\frac{4}{3}m^2$, 0

5. $5x - 3$, 0 **6.** $4p - 2$, 0 **7.** $y + 2$, 3 **8.** $3x$, $-14x^2 + 7x$

9. $3x^3 - 2x^2 + 4x + 1$, 0 **10.** $5m - 3$, $10m + 8$

Practice Set 10.2

1. $y + 6$, 0 **2.** $p + 4$, -17 **3.** $4x^2 + 18x + 75$, 300

4. $m^2 + m + 1$, 10 **5.** $x^2 + x - 5$, $x - 2$

6. $a - 1$, $a^2 + a - 1$ **7.** $x^3 - x^2 - \frac{x}{4} - \frac{29}{16}$, $\frac{-13}{16}$

D3HCQB